



DECUS

PROGRAM LIBRARY

DECUS NO.	8-485
TITLE	GEOMETRIC DATA TRUNCATION FOR FOURIER TRANSFORM PROGRAMS
AUTHOR	E. A. Barnhardt
COMPANY	Southwestern at Memphis Memphis, Tennessee
DATE	
SOURCE LANGUAGE	PAL III

1. Program Title: Geometric Data Truncation for Fourier Transform Programs
2. Abstract: This program is intended for use as a truncating-weighting subroutine in conjunction with a Fourier Transform program. The application of a weighting function to the data record before the application of a Fourier transform program reduces the spread in frequencies which results from the transformation of a finite record. This procedure is often called apodization in that it removes the side lobes in the transformed record that results from a rectangular data window.
3. Requirements:
 - 3.1 Storage: The subroutine occupies 115(8) locations and is relocatable at the time of assembly.
 - 3.2 Hardware: PDP-8
 - 3.3 Software: The user must have a mainline program for gathering and storing in serial order the data before the application of this truncation algorithm. Following the weighting of the data the transform must be applied. The real series must be stored in serial order and without gaps. The length must be of the form 2^N , just as required by the FFT. Additionally, the data must be in signed 11 bit, fixed-point format.
4. Usage: The program is intended to be assembled along with the mainline program. It is to be called after the data are gathered and before the actual FFT. The weighted data are replaced in their original locations with the original contents being lost. The subroutine is called with the number of data points in the AC and the address of the first point in CALL +1. Return is to CALL +2.
5. Discussion: When performing the discrete Fourier transform on a finite record in the time domain an inherent distortion results in the frequency domain. The FFT gives the power spectral density of an observed signal. Denoting Fourier transforms by ($\overline{\quad}$) and convolutions by (*),

$$A(v) = \overline{A(t)} \quad \& \quad F(v) = \overline{F(t)}$$

where

$$A(v) = \int_{-\infty}^{\infty} A(t) e^{-i2\pi vt} dt$$

If $F(t)$ is the temporal representation of the observed signal, the power spectral density is given by

$$F(v) = \left| \int_{-\infty}^{\infty} F(t) e^{-i2\pi vt} dt \right| = \left| \overline{F(t)} \right|$$

The data window here extends from $-\infty$ to $+\infty$ and weights each datum equally, i.e., $A(t) = 1$ for $-\infty \leq t \leq +\infty$.

The multiplication of two functions in the time domain is equivalent to convolving the transforms of the two functions in the frequency domain.

$$\begin{aligned} \overline{A(t) \cdot F(t)} &= \overline{A(t)} * \overline{F(t)} = A(v) * F(v) \\ &= \int_{-\infty}^{\infty} A(v - v') F(v) dv' \end{aligned}$$

For the case of an infinite data window

$$A(v) = \overline{A(t)} = \int_{-\infty}^{\infty} 1 \cdot e^{-i2\pi vt} dt = \delta(v - v')$$

Hence the power spectral density

$$F(v) = \left| \int_{-\infty}^{\infty} F(t) e^{-2i\pi vt} dt \right| = \left| \int_{-\infty}^{\infty} \delta(v - v') F(v) dv' \right|,$$

is the true power spectrum.

Infinite records are impossible and the observations of the temporal signal must be terminated. This limiting of $F(t)$ is truncation.

The most common form of truncation is the rectangular data window. Observations are made for a certain period of time and all information before and after this period is neglected. This gives rise to an $A(t)$ that is a rectangle of unit height. The Fourier transform, $A(v) = \frac{\sin \pi v}{\pi v}$, is a function with a finite spread and

smaller side lobes, (see Fig 1A). The true spectrum is then convolved with a function which spreads each frequency into several frequencies.

Several other truncation functions have been used. These functions reduce the spread of the central peak

and/or remove the side lobes. Two of these are the triangular function with $A(v) = \left(\frac{\sin \pi v}{\pi v}\right)^2$ (see Fig. 1B).

and the Gaussian function with $A(v) = e^{-\pi v^2}$, (see Fig. 1C). The application of these functions involves a degree of data handling that may be overly time consuming in order to obtain a clean power density spectrum.

This program involves a geometric truncation which requires no multiplications, or divisions, and yet gives a smooth truncated signal to be transformed. In the frequency domain the function to be convolved with the true spectral function has no side lobes and a minimal width.

6. Description: The data record is first divided into 16 blocks. The binary representation of each datum within a block is shifted to the right a number of times dependent upon the position of the block in the data record. The components of the blocks would be shifted 7, 6, 5, ..., 1, 0, 0, 1, 2, ..., 6, 7 places, resulting in multiplications of the components by $1/128, 1/64, 1/32, \dots, 1/2, 1, 1, 1/2, 1/4, \dots, 1/64, 1/128$ respectively. These factors are indicated by the horizontal bars in Fig. 2.

The truncation function when smoothed to its average value is the hyperbolic secant, $\text{sech}(\pi t)$. The geometrical function has a number of discontinuities but at points of discontinuity the Fourier transform integral converges to the average

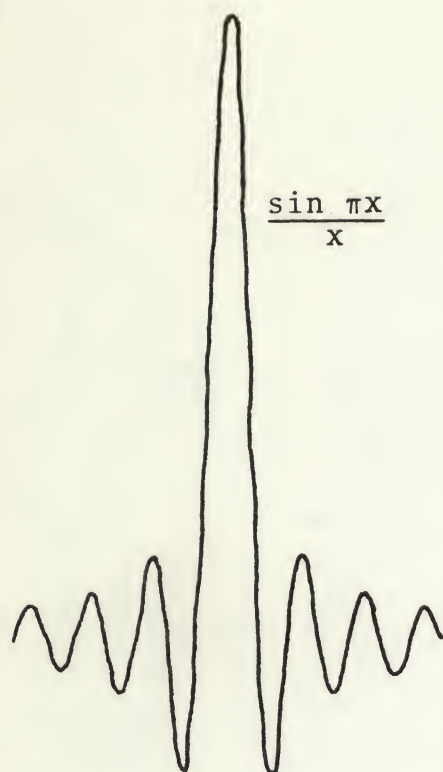
$$\lim_{\epsilon \rightarrow 0} \frac{1}{2} [A(t + \epsilon) + A(t - \epsilon)]$$

of the right and left hand limits.

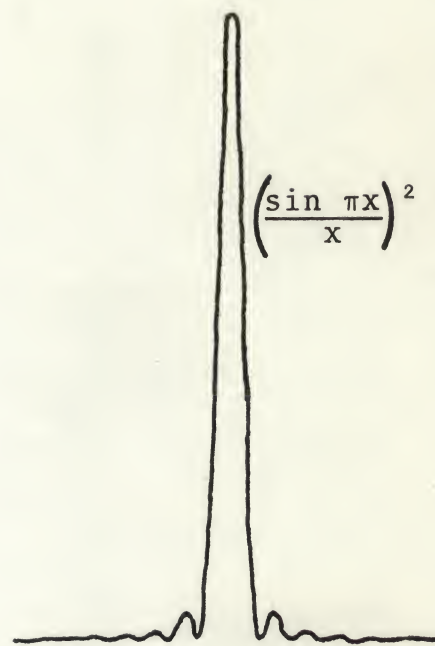
The Fourier transform of $\text{sech}(\pi t)$ is $\text{sech}(\pi v)$, (see Fig. 1D). So the function to be convolved with the true spectrum has no side lobes and minimal width.

7. Execution Time: The time required to weight a data record is $(100 \mu s) \times (\text{data record length})$.
For example, a 512(10) point record length can be weighted in 0.0512 seconds.
8. User Modification: The user who intends to transform data records of a fixed length may wish to shorten the subroutine by calculating and storing some of the constants which would otherwise be recalculated and stored at each execution time.

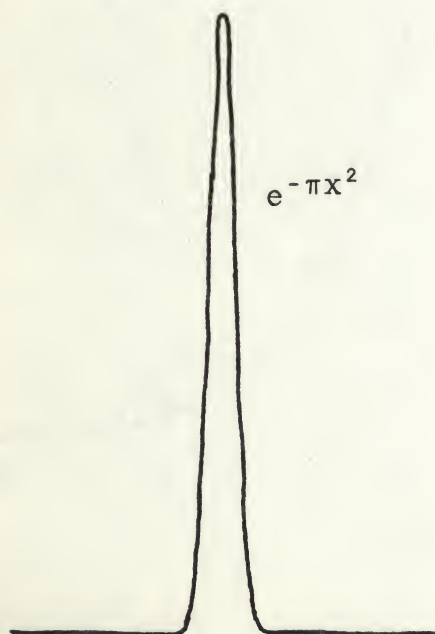
9. Listing: A listing is below.
10. Logic Flow Chart: A logic structure flow chart follows the listing.
11. References:
 - (1) Bergland, G. D. "A Guided Tour of the Fast Fourier Transform", IEEE Spectrum, 6, 41 (1969).
 - (2) Bingham, C., Godfrey and Tukey, "Modern Techniques of Power Spectrum Estimation." IEEE Trans. Audio Electroacoustics, AU-15, 56 (1967).



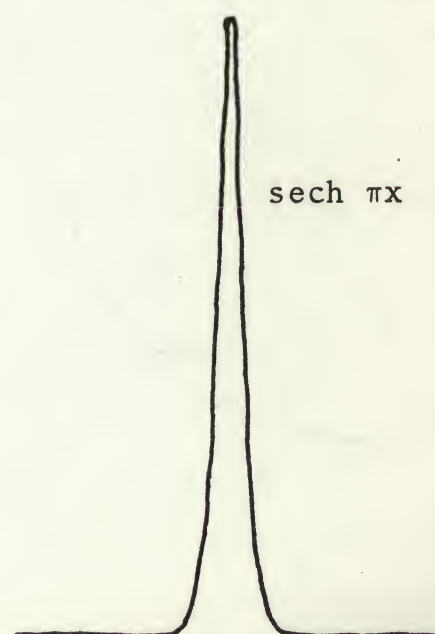
1A



1B



1C



1D

Figure 1

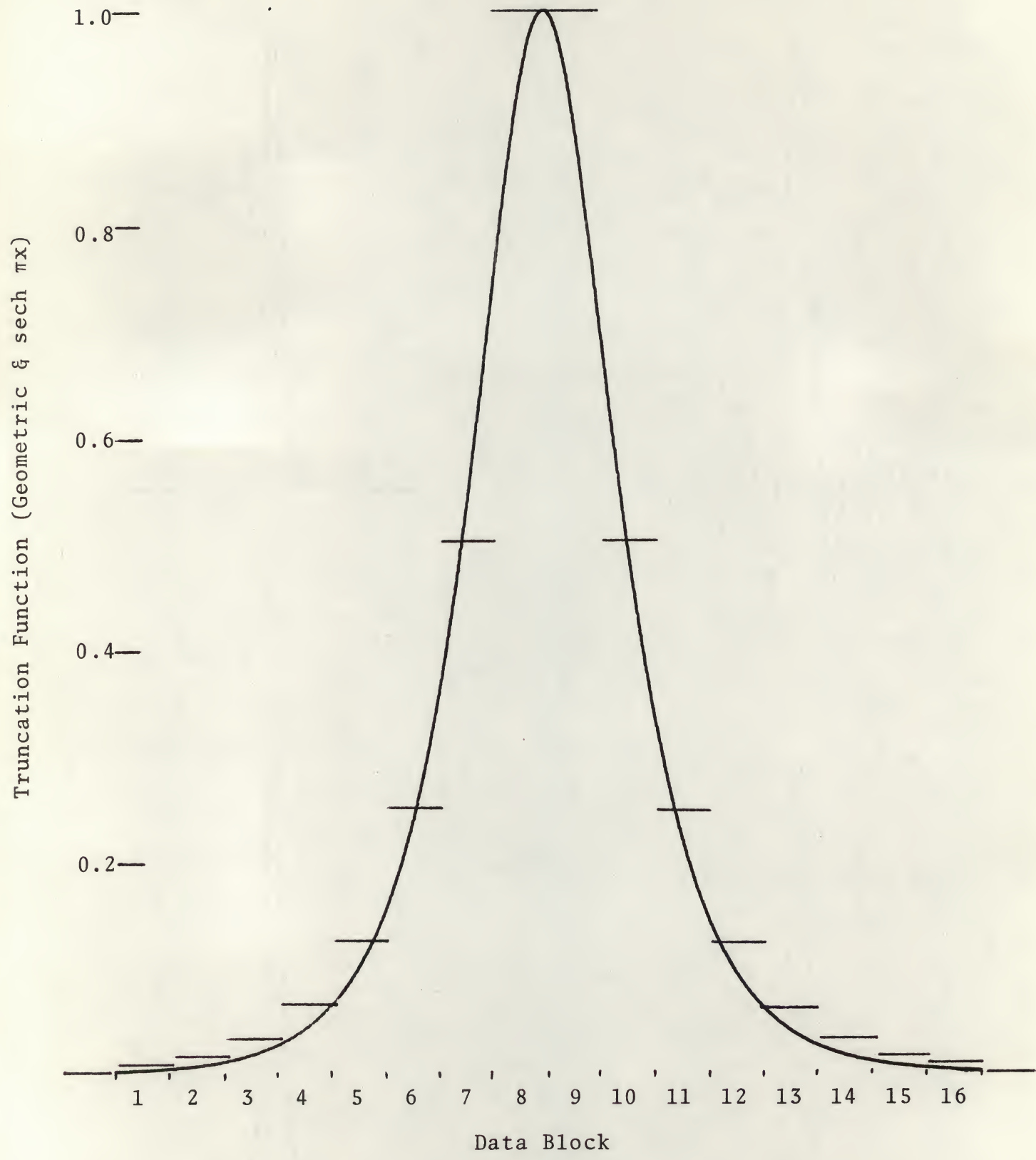


Figure 2

/GENERALIZED AFODIZATION FUNCTION
 /E.A. RAENHARDT, 30 SEPTEMBER 1971
 /FOR DATA OF LENGTH 2*N, THAT IS 1024, 512, 256, ETC
 /RUNNING TIME, 100 MICROSEC TIMES DATA LENGTH
 /DATA RESTORED TO ORIGINAL LOCATION.
 /CALL WITH NUMPFF OF DATA POINTS IN AC AND ADDRESS
 /OF FIRST POINT IN CALL+1, RETURN IS TO CALL+2
 /PROGRAM OCCUPIES 115(8) LOCATIONS
 *4600

4600	0000	APOD,	C	
4601	3313		DCA PTS	/NUMBER OF DATA POINTS (8)
4602	1600		TAD I AFOD	
4603	3311		DCA ADD1	/ADDRESS OF FIRST DATUM
4604	2200		ISZ AFOD	/SET RETURN TO CALL +2
4605	1313		TAD PTS	
4606	7012		FTF	
4607	7012		FTF	
4610	3302		DCA TEMP2	/LENGTH OF ONE DATA BLOCK
4611	1302		TAD TEMP2	
4612	7041		CIA	
4613	3306		DCA CNTR3	/MINUS BLOCK LENGTH
4614	1313		TAD PTS	
4615	1306		TAD CNTR3	
4616	1306		TAD CNTR3	
4617	7041		CIA	
4620	3307		DCA CNTR4	/ACTUAL # OF POINTS TO BE SHIFTED
4621	1313		TAD PTS	
4622	7010		RAF	
4623	1311		TAD ADD1	
4624	1302		TAD TEMP2	
4625	3312		DCA ADD2	/ADDRESS OF 2ND BLOCK OF 2ND HALF
4626	1314		TAD M7	
4627	3303		DCA CNTR0	/MINUS 7, MINUS 6, MINUS 5, ...
4630	1306	LOOP1,	TAD CNTR3	
4631	3310		DCA CNTR5	
4632	1711	SLOOP1,	TAD I ADD1	/GET DATUM
4633	4265		JMS SPSR	/SHIFT
4634	3711		DCA I ADD1	/DEPOSIT SHIFTED DATUM
4635	2311		ISZ ADD1	/UPDATE ADDRESS
4636	2307		ISZ CNTR4	/ALL COMPLETE?
4637	2306		ISZ CNTR3	/NO, BLOCK COMPLETE?
4640	5232		JMP SLOOP1	/NO, GET NEXT DATUM
4641	2303		ISZ CNTR0	/YES, REDUCE SHIFT OR GO TO 2ND HALF
4642	5230		JMP LOOP1	/DO NEXT BLOCK
4643	7300		CLA CLL	/SECOND HALF INITIALIZATION
4644	7001		IAC	
4645	7041		CIA	
4646	3303		DCA CNTR0	/MINUS 1, MINUS 2, MINUS 3, ...
4647	1306	LOOP2,	TAD CNTR3	
4650	3310		DCA CNTR5	
4651	1712	SLOOP2,	TAD I ADD2	/GET DATUM
4652	4265		JMS SFSP	/SHIFT
4653	3712		DCA I ADD2	/DEPOSIT SHIFTED DATUM
4654	2312		ISZ ADD2	/UPDATE ADDRESS

4655	2307	ISZ CNTR4	/ALL COMPLETE?
4656	5260	JMP SKIP	/NO, SKIP
4657	5600	JMP I AFOD	/YES, RETURN
4660	2310	SKIP, ISZ CNTR5	/BLOCK COMPLETE?
4661	5251	JMP SLOOP2	/NO, GET NEXT DATUM
4662	1303	TAD CNTR0	/YES, INITIALIZE FOR NEXT BLOCK
4663	7041	CIA	
4664	5247	JMP LOOP2	
4665	0000	SFSE, 0	/SHIFT SUP. (MODIFIED DEC-08-PMJA-D)
4666	3301	DCA TMEF1	/STORE NUMPFP
4667	1303	TAD CNTR0	/GET SHIFT COUNTER
4670	3315	DCA CNT	/STORE FOR LOCAL USE
4671	1301	TAD TMEF1	
4672	7100	LOOP3, CLL	
4673	7510	SFA	
4674	7020	CML	
4675	7010	FAD	
4676	2315	ISZ CNT	
4677	5272	JMP LOOP3	
4700	5665	JMP I SFSE	
4701	0000	TEMP1, 0	
4702	0000	TEMP2, 0	
4703	0000	CNTR0, 0	
4704	0000	CNTR1, 0	
4705	0000	CNTR2, 0	
4706	0000	CNTR3, 0	
4707	0000	CNTR4, 0	
4710	0000	CNTR5, 0	
4711	0000	ADD1, 0	
4712	0000	ADD2, 0	
4713	0000	PTS, 0	
4714	7771	M7, 7771	
4715	0000	CNT, 0	

ADD1	4711
ADD2	4712
AFOD	4600
CNT	4715
CNTR0	4703
CNTR1	4704
CNTR2	4705
CNTR3	4706
CNTR4	4707
CNTR5	4710
LOOP1	4630
LOOP2	4647
LOOP3	4672
M7	4714
PTS	4713
SKIP	4660
SLOOP1	4632
SLOOP2	4651
SFSE	4665
TMEF1	4701
TEMP2	4702

